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Method for signal decomposition and denoising based on nonuniform cosine-modulated filter banks

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Abstract

In this paper, a novel method for signal decomposition and denoising is proposed based on a nonuniform filter bank (NUFB), which is derived from a uniform filter bank. With this method, the signal is firstly decomposed into M subbands using a uniform filter bank. Then according to their energy distribution, the corresponding consecutive filters are merged to compose the nonuniform filters. With the resulting NUFB, the signal can be readily matched and flexibly decomposed according to its power spectrum distribution. As another advantage, this method can be used to detect and remove the narrow-band noise from the corrupted signal. To verify the proposed method, a simulation of extracting the main information of an audio signal and removing its glitch is given.

Keywords: Signal decomposition; Denoising; Nonuniform filter bank; Narrow-band noise

1. Introduction

Signal decomposition and denoising are two essential aspects in the fields of audio signal and image processing. The work on signal decomposition by using uniform filter banks (FBs) is well published. However, in some applications, such as audio coding and subband adaptive filtering, nonuniform frequency spacing that matches the critical bands is much more appreciated. Hence, some attentions have been paid to the methods for the decomposition with nonuniform filter banks (NUFBs) [1–5]. In Ref. [5], Xie proposed an approach by using a recombination NUFB, where certain channels of a uniform FB are combined by sets of synthesis filters of uniform FBs with a smaller number of channels according to the power spectrum distribution of the signal. However, the proposed system has a long delay due to the two-stage architecture. For denoising, Tazebay [6] proposed a method for removing narrow-band noise in the spread spectrum system, in which tree structure with 2- or 3-channel uniform FBs being employed is used to detect and remove the noise adaptively. The work is further improved in Ref. [7] by using low-delay FBs in the tree structure. Although it is possible to reduce the system delay to a certain degree, the reduction is limited due to multistage operation. In addition, the implementation complexity of the system is still high. While the wavelet-based [8,9] and wavelet-packet based [10] denoising methods are widely used to remove the white gaussian noise, they are not suitable for the reduction of the narrow-band noise. In terms of the joint application of decomposition and denoising, Mitrovski proposed a method based on quadrature mirror filters to pre-process nuclear medicine images [11]. Due to the fact that the FBs used are only of two bands, the decomposition is not flexible.

In this study, a simple and effective method for signal decomposition and denoising is proposed. The decomposition is carried out nonuniformly according to the power spectrum distribution of the signal and the noise to be

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removed is of narrow-band. The proposed method is based on an NUFB which is obtained by merging the corresponding filters of a uniform cosine-modulated filter bank (CMFB) under certain requirements for the energy distribution of the signal. This method performs efficiently when the power spectrum of the signal is distributed concentratedly. Moreover, if the signal is corrupted by a narrow-band noise in the region where there is no main information, the noise can be removed easily by the NUFB. In our work, by employing the Parks–McClellan algorithm which is optimal in the design of filters in the minimax sense, the high quality of uniform FBs for the nonuniform merging can be obtained with low design effort. Simulation for audio signal is given to verify that the proposed method is capable of achieving good results in extracting the main information of the signal and removing the narrow-band noise, with detailed analyses of the method from the viewpoint of time and computation complexity. Finally, the denoising result obtained by the wavelet-packet-based denoising method is given to show that these conventional methods are not good at suppressing the narrow-band noise.

2. Design of nonuniform cosine-modulated filter banks

In this section, we focus on the design of the optimal prototype filter for uniform CMFBs. Then, a brief review of the merging method for NUFB is given.

2.1. Design of optimal prototype filters

In the uniform CMFB, the analysis and synthesis filters $H_l^u(z)$ and $F_l^u(z)$ are all generated by cosine-modulating a prototype filter $P_0(z)$. Their corresponding impulse responses are given by

$$\begin{aligned} h_l^u(n) &= 2p_0(n) \\ &\times \cos\left(\frac{\pi}{M}(l+0.5)\cdot\left(n-\frac{N}{2}\right)+(-1)^l\frac{\pi}{4}\right) \\ f_l^u(n) &= 2p_0(n) \\ &\times \cos\left(\frac{\pi}{M}(l+0.5)\cdot\left(n-\frac{N}{2}\right)-(-1)^l\frac{\pi}{4}\right) \\ &0 \leqslant n \leqslant N, 0 \leqslant l \leqslant M-1 \end{aligned} \tag{1}$$

where N is the order of $P_0(z)$. If the prototype filter is of linear phase ($p_0(n) = p_0(N-n)$), $f_l^u(n)$ is the time reversed version of $h_l^u(n)$. Thus, as shown in Ref. [12], the transfer function $T(e^{j\omega})$ can be expressed as

$$\begin{aligned} MT(e^{j\omega}) &= \sum_{l=0}^{M-1} H_l^u(e^{j\omega})\cdot F_l^u(e^{j\omega}) \\ &= e^{-j\omega N}\sum_{l=0}^{M-1} |H_l^u(e^{j\omega})|^2 \end{aligned} \tag{2}$$

in the frequency domain. It is obvious that $T(e^{j\omega})$ is of linear phase and the system is free from phase distortion. From the above analysis, we can conclude that the significant aliasing distortion between adjacent channels has been canceled by nature. Thus, for an NPR filter bank, only the amplitude distortion should be considered.

As shown in Eq. (2), if the stopband attenuation of $H_l^u(z)$ is sufficiently high, $|T(e^{j\omega})|$ will be as nearly flat as the magnitude response of the individual $H_l^u(z)$ in the passband. And if the sum of the squared magnitude responses of the adjacent two analysis filters is close to unity in their transition band, $|T(e^{j\omega})|$ will be approximately flat. In the CMFB, to meet the requirements described above, it is sufficient to impose the following constraint on the prototype filter $P_0(z)$

$$|P_0(e^{j\omega})|^2 + |P_0(e^{j(\omega-\frac{\pi}{M})})|^2 = 1, \quad 0 < \omega < \frac{\pi}{M} \tag{3}$$

Moreover, the requirement in the transition band would be met if we force the response of $P_0(z)$ in its transition band to follow a cosine function.

The Parks–McClellan algorithm is employed in our design of optimal prototype filters because of its simplicity and good performance. Thus, the magnitude response is completely specified and used as the design target. In addition, the stopband attenuation of the resulting filter bank is significantly higher than those obtained by the traditional methods.

2.2. Merging method for nonuniform cosine-modulated filter banks

In this section, we present a design method for NUFBs with integer sampling factors. As the nonuniform FB shown in Fig. 1(a), the decimation factors n_k are not equal. $H_k(z)$ and $F_k(z)$, $k = 0, 1, \cdots, K-1$, are the analysis and synthesis filters, respectively. To simplify the analysis, we assumed that only the sampling factor of one branch in

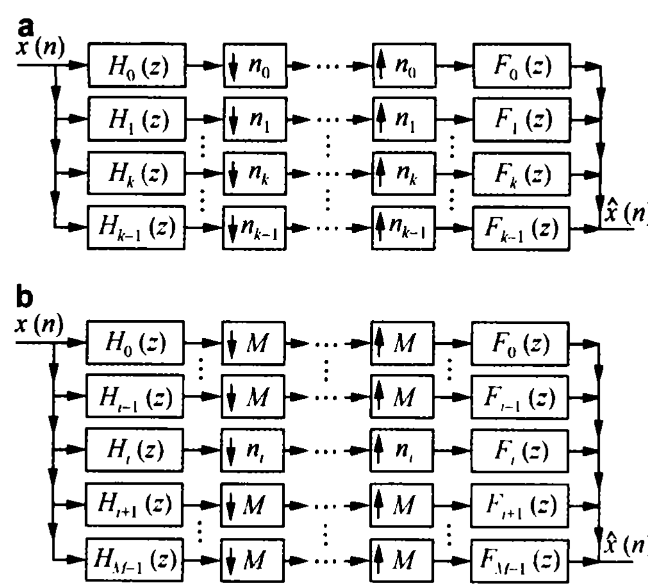

Fig. 1. (a) K-channel NUFB, (b) NUFB with the decimator of the ith channel different from others.

the NUFB, say, the ith branch, is different from the others, as shown in Fig. 1(b). We can obtain the nonuniform analysis and synthesis filters from the uniform ones of the CMFB under the following relationship [13]

$$\begin{aligned} &H_k(z) = H_k^u(z), F_k(z) = F_k^u(z), \\ &\qquad \text{for} \quad 0 \leqslant k \leqslant i-1, 0 \leqslant i \leqslant M-1 \\ &H_i(z) = \sum_{j=0}^{p_i-1} H_{i+j}^u(z), F_i(z) = \frac{1}{p_i}\sum_{j=0}^{p_i-1} F_{i+j}^u(z) \\ &H_k(z) = H_{k+p_i-1}^u(z), F_k(z) = F_{k+p_i-1}^u(z), \\ &\qquad \text{for} \quad i+1 \leqslant k \leqslant K-1 \end{aligned} \quad (4)$$

where $p_i = M/n_i$, and M is the least common multiple (lcm) of $n_0, n_1, \cdots, n_{K-1}$.

With the merging method, the properties of the uniform CMFBs are mostly preserved in the resulting NUFB. In this paper, we use uniform CMFBs with optimal prototype filter to design the NUFB; thus, the high quality of the NUFB can be obtained with low design effort.

2.3. Design example of the NUFB

In this section, a design example of a 3-channel NUFB with sampling factors 4, 4, 2 is given, where $n_0 = n_1 = 4$, $n_2 = 2$ and $M = 4$, $p_i = M/n_i = 2 (i = 2)$. The structure of this system is shown in Fig. 2(a). Firstly, we design the optimal prototype filter $p_0(n)$ with the length of 64. Secondly, the 4-channel uniform CMFB is obtained by modulating the optimal filter. Finally, the analysis and synthesis filters of the NUFB are obtained based on this CMFB using Eq. (4). The magnitude responses of analysis filters are shown in Fig. 2(b). The maximum values of the amplitude and aliasing distortion are 2.155×10^{-3} and 9.193×10^{-6}, respectively. And the stopband attenuation is 93 dB.

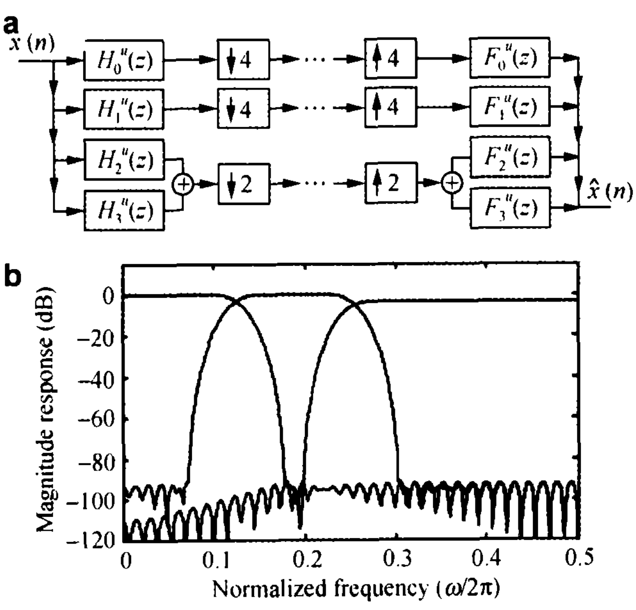

Fig. 2. NUFB with sampling factors {4,4,2}. (a) Structure of the system; (b) magnitude responses of the analysis filters.

Under the same design specifications such as filter length and stopband cutoff frequency, the amplitude distortion given in Ref. [13] is 1.094×10^{-3}, which is almost the same as that in our work. However, the aliasing distortion is 7.943×10^{-4}, which is larger than that obtained by our proposed method, and the stopband attenuation by using our approach is nearly 31 dB higher than that in Ref. [13] (93 dB versus 62 dB). More importantly, the analysis and synthesis filters in our design are capable of achieving the comparable performance without nonlinear optimization.

3. Method for signal decomposition and denoising based on NUFBs

In this section, we give a simple approach to signal decomposition and denoising based on NUFBs. It is suitable for the case that the power spectrum of the signal is distributed concentratedly.

3.1. Idea of nonuniform signal decomposition and denoising

3.1.1. Nonuniform signal decomposition

For the design of the NUFB as described in Section 2, an M-channel uniform CMFB should be designed first. The choice of the number M depends on the required decomposition precision. With the analysis filters of the resulting CMFB, the signal can be decomposed into M subbands uniformly. Obviously, if we predefine N thresholds, those uniform bands can be classified into $N+1$ levels. Those predefined thresholds are set according to the decomposition requirement and magnitude response of the input signal, instead of the computation method proposed by Donoho [8].

To simplify the illustration, we take the decomposition with one threshold as an example. For each uniform band, if its magnitude is smaller than the threshold, it will be referred to as a low energy band; otherwise, it is a high energy band. With the threshold, the signal is decomposed into two levels: low energy level and high energy level, where the low (high) energy level consists of low (high) energy bands. For easy description, the energy characteristics of the uniform bands are denoted by

$$\begin{cases} 1 & \text{High energy subband,} \\ 0 & \text{Low energy subband.} \end{cases}$$

Obviously, since the energy of the signal considered is distributed concentratedly, the elements ‘1’ and ‘0’ are also distributed concentratedly. Here, we take $M = 12$ as an example. If the energy characteristics of the twelve uniform subbands are [001111110000], we merge the uniform analysis filters to form the nonuniform ones using Eq. (4), Uniform $H_0^u\ H_1^u\ H_2^u\ \cdots\ H_7^u\ H_8^u\ \cdots\ H_{11}^u$

$$[\underbrace{0\ 0}\ \underbrace{1\ 1\ 1\ 1\ 1\ 1}\ \underbrace{0\ 0\ 0\ 0}]$$

Nonuniform H_0 H_1 H_2

Thus, a 3-channel NUFB is obtained. With the analysis filters of the obtained NUFB, the input signal can be flexibly decomposed into high energy level and low energy level in the frequency domain.

3.1.2. Application to denoising

If the input signal is corrupted by a narrow-band noise in its low energy bands, the noise can be removed based on the idea of decomposition. The corrupted signal $x_c(t)$ can be expressed as

$$x_c(t) = x_o(t) + n(t) \tag{5}$$

where $x_o(t)$ is an original signal and $n(t)$ a narrow-band noise.

As described above, an M-channel uniform CMFB should be designed first. The difference is that the choice of M depends on the bandwidth of the narrow-band noise and the design complexity of FBs. In order to remove the noise while retaining the characteristics of the original signal as much as possible, the bandwidth of each uniform filter should be equal to or less than that of the narrow-band noise. The larger the M is, the more efficient the denoising method is. However, the design complexity of FBs increases with the increasing number M.

Let the corrupted signal $x_c(t)$ pass through the uniform CMFB. Since the narrow-band noise $n(t)$ appears in the low energy bands of the signal, there must be one or two '1's distinguishing the noise from other low energy bands denoted by '0's. For example, if the energy characteristics of the uniform subbands are [001111110010], the noise $n(t)$ must be located in the 10th subband of the uniform CMFB. Therefore, we take the following operation to obtain the required NUFB

Uniform: $H_0^u\ H_1^u\ H_2^u\ \cdots\ H_7^u\ H_8^u\ H_9^u\ H_{10}^u\ H_{11}^u$

$$[\underbrace{0\ 0}\ \underbrace{1\ 1\ 1\ 1\ 1\ 1}\ \underbrace{0\ 0}\ 1\ \ 0]$$

Nonuniform H_0 H_1 H_2 H_3 H_4

Each subband signal obtained by the nonuniform analysis filters can be processed in different ways. For the noise detected, one simple but effective method is to delete the 3rd subband containing noise in the reconstruction of the signal. The reconstructed signal is with little information loss.

Since the subbands which contain the noise are discarded in the reconstruction procedure, the PR of the signal is not possible. Hence, an NPR NUFB is used here instead of a PR system.

3.2. Simulation

In this section, an example is given to illustrate the capability of the proposed method to extract the main information of the signal and remove the narrow-band noise, followed by the analysis of the time and computation complexity. Furthermore, we give the denoising result by the wavelet-packet-based denoising method, which shows that this kind of method is not suitable for removing the narrow band noise.

The audio signal "chirp" in Matlab is taken to demonstrate the idea of signal decomposition and denoising. The magnitude response of the signal "chirp" is shown in Fig. 3(a). It can be seen that the power spectrum is distributed concentratedly and the threshold can be chosen as 5. Fig. 3(b) plots the corrupted signal with a narrow-band noise.

As mentioned above, the bandwidth of each uniform filter is equal to or less than that of the noise. In this example, the bandwidth of the noise is $2\pi/12$. Therefore, we choose the number of channel M as 24. Note that the length of the filter is chosen according to the desired performance and the system delay. Here, we choose the length of 141 as an example. By using the method of the uniform CMFB as described in Section 2.1, we get the uniform NPR CMFB. With the predefined threshold, the noise and the main components (high energy bands) are distinguished from each other. Fig. 3(c) shows the frequency responses of the analysis filters of the 24-channel uniform filter bank, where the subbands for noise and the main component are indicated. With the corresponding uniform filters, we get the desired NUFB by the merging method as mentioned in Section 2.2. The frequency responses of the nonuniform filters are shown in Fig. 3(d). The structure of the NUFB is illustrated in Fig. 4, which shows that (i) the four nonuniform filters are generated by merging uniform filters H_0–H_3, H_4 and H_5, H_6–H_{11}, H_{12} and H_{13}; (ii) the decimation factors of the four channels are 6, 12, 4, 2 (say 24/4, 24/2, 24/6 and 24/12); (iii) the input signal can be decomposed nonuniformly into the subbands with different levels of energy and they can be further processed in different ways. Here, as an example, we remove the subband with noise signal in the processing. Fig. 3(e) shows the frequency spectrum of the reconstructed signal without the noise subband. From Fig. 3(a), (b) and (e), which show the original, the corrupted and the reconstructed signals, respectively, we can easily see that the output signal is recovered by using our proposed method. And, it is obvious that the proposed method can extract the main information of the signal effectively.

For the corrupted signal with signal to noise ratio (SNR) of 19.18 dB, the reconstructed signal obtained by the NUFB as shown in Fig. 4 achieves an SNR of 34.08 dB, which is improved by 14.90 dB. The simulation shows that the proposed method can decompose the signal with high flexibility and remove the narrow-band noise easily. Next, the time complexity and computation complexity are analyzed.

Both FFT and convolution are the basic operations for our method. As is known, the time complexity of FFT is $n \log n$, where n is the number of points in FFT; and the

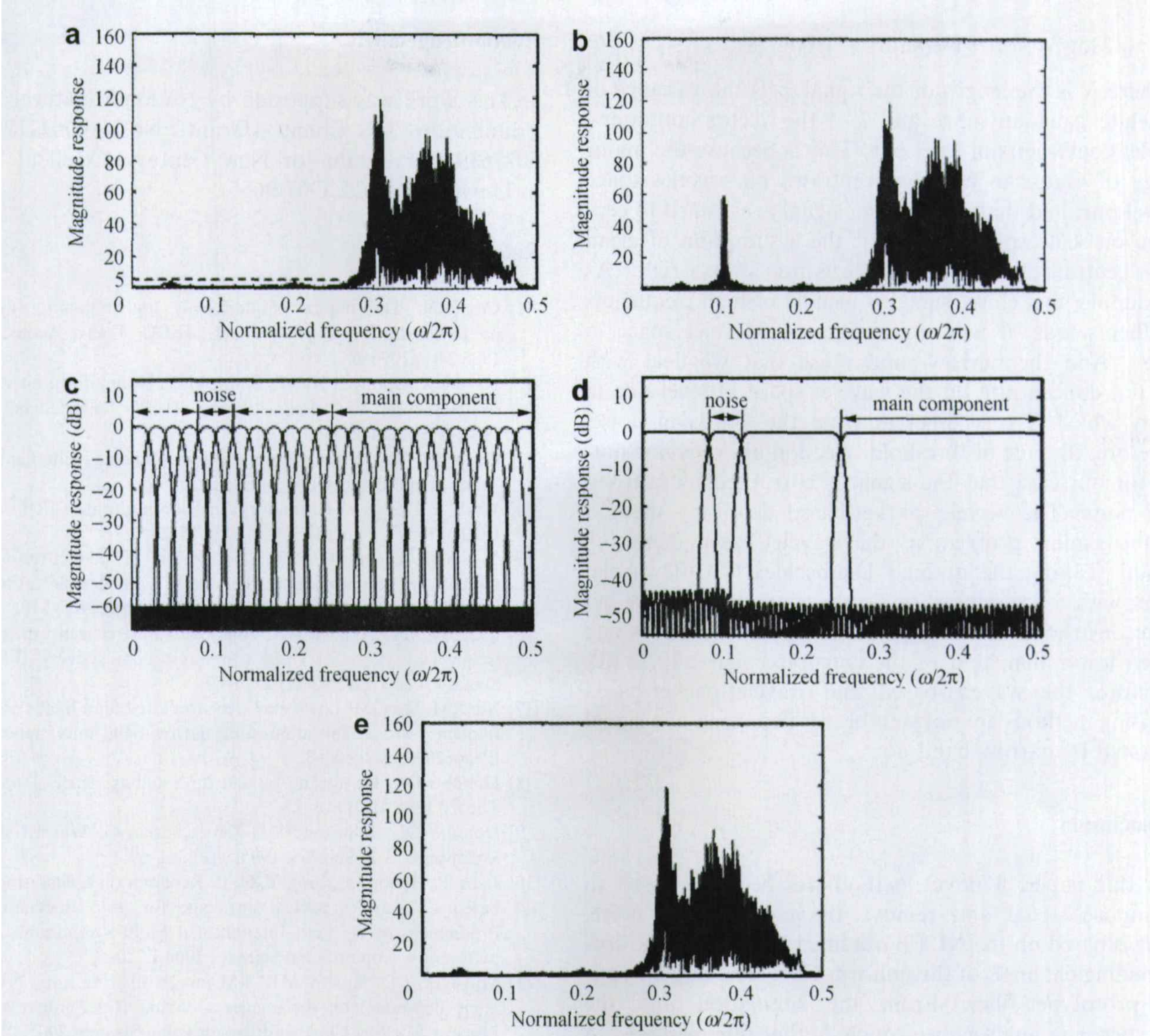

Fig. 3. Magnitude responses of (a) the original signal $x_0(n)$, (b) the corrupted signal $x_c(n)$; (c) the analysis filters of the uniform CMFB; (d) the analysis filters of the resulting NUFB; (e) the reconstructed signal.

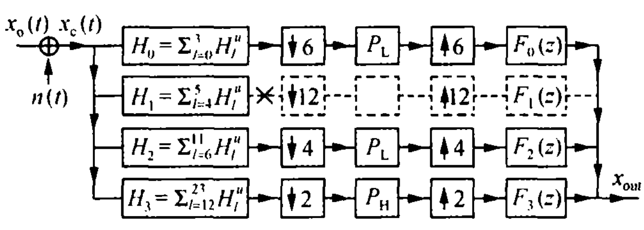

Fig. 4. System structure of the method in the example ($M = 24$, P_L: process for low energy level; P_H: process for high energy level).

complexity of convolution is about L_sL, where L_s and L are the lengths of signal and filter, respectively. Thus, for our method as described in Section 3.1, the total time complexity is $O(n \log n + LL_sM_u + L^2M_n)$, where M_u and M_n are the numbers of channels for the uniform FB and NUFB, respectively.

The computation complexity is measured by the numbers of the required additions per unit time (APUs) and multiplications per unit time (MPUs). In this study, the NUFB shown in Fig. 1(a) is implemented by using polyphase decomposition, as mentioned in Refs. [7] and [12]. The numbers of APUs and MPUs can be calculated as APUs $:\sum_{k=0}^{K-1}(L-1)/n_k$ and MPUs $:\sum_{k=0}^{K-1}L/n_k$, where L is the length of the filter. In this study, the length of the filter is 141 and the down-sampling factors n_k are 6, 4 and 2 (the second channel with down-sampling factor 12 as shown in Fig. 4 is deleted). Therefore, for NUFB shown in Fig. 4, the computation complexity is APUs $\approx$ MPUs ≈ 132.

For comparison, we will analyze whether the popular wavelet-based and wavelet-packet-based denoising methods can be used to solve the problem of denoising the narrow-band noise. Actually, denoising methods based on wavelet show exciting results for the denoising of a signal buried in white Gaussian noise through a threshold. The choices of threshold of these methods are mainly based on the consideration of the variance of wavelet coefficients in decomposition level one [8]

$$T = \sigma\sqrt{2\log N} \quad (\sigma = \text{median}(|d|)/0.6578) \tag{6}$$

where N is the length of the signal, σ is the variance of the white gaussian noise and d is the vector containing wavelet coefficients in level one. This is because the major energy of Gaussian noise concentrates on wavelet space at level one, and that of the clean signal is assumed to centralize on scale space. However, the assumption of clean signal centralizing on scale space is not always right. As an example, the 'chirp' signal is usually of high frequency. In other words, it will concentrate on wavelet space in energy. And the narrow-band noise that we deal with may not concentrate on the wavelet space at level one in energy which is very different from the Gaussian noise. Therefore, the rule of threshold selection above is not suitable for our case that the signal is corrupted by narrow-band noise. The wavelet-packet-based denoising method has the similar problem as the wavelet-based denoising method. Taking the optimal Daubechies (db16) wavelet packet with 5 decomposition levels as an example, we get the reconstructed signal with an SNR of 5.63 dB, which is even lower than that of the corrupted signal 19.18 dB. Therefore, the wavelet-based and wavelet-packet-based denoising methods are not suitable for denoising of a signal corrupted by narrow-band noise.

4. Conclusion

In this paper, a novel method has been proposed to decompose signal and remove the narrow-band noise, which is based on the NUFB obtained by merging the corresponding channels of the uniform CMFB having an optimal prototype filter. From the simulation and the corresponding analysis, we conclude that this method can decompose signal with high flexibility and to effectively remove the narrow-band noise which cannot be well removed by the wavelet-based and wavelet-packet-based denoising methods.

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